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Engineering Notes

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Design Technique for Stuck-On Thruster Δv Minimization for an Asymmetric Spacecraft

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Introduction

A STUCK-ON thruster is a serious event for any spacecraft. Examples have included the Gemini 8 crewed spacecraft[1] in 1966, where the result was early termination of the mission; the Clementine lunar/asteroid probe,[2] which was lost as a result of a software problem that led to thruster firing until propellant depletion, spinning the spacecraft up to over 80 rpm; and the Wide-Field Infrared Explorer (WIRE) spacecraft,[3] which was spun-up to around 60 rpm shortly after launch by the inadvertent boiloff of its solid hydrogen coolant. This applied a body-fixed torque to the satellite, dynamically equivalent to a stuck-on thruster. Although WIRE was eventually stabilized, the complete absence of cryogen led to the loss of its science mission.

The dynamics of a vehicle with a stuck-on thruster have been addressed for the special case of an axisymmetric body,[4,5] or one that is nearly so.[6,7] They were also addressed by the present authors[8] for the case of a vehicle that is nominally spherically symmetric, but is in reality slightly perturbed from this ideal. Results have also been obtained[4,9] for the case of an asymmetric vehicle (moments of inertia all distinct) that is excited by a torque precisely about one of its principal axes. This closely approximates, for instance, the problem of a three-axis stabilized spacecraft that suffers a stuck-on attitude thruster. However, the principal axes of a real vehicle will not actually be aligned precisely with its reference body axis system. Consequently, the torque produced by a stuck-on thruster will only be approximately aligned with a principal axis. This imperfect alignment can lead to great differences in the response of the system when the torque is nominally about the intermediate axis. In particular, such a stuck-on thruster can now, under some circumstances, give rise to a large Δv, leading to uncommanded orbit changes and a possible collision hazard; this never occurs in the idealized case of perfect torque alignment. This Note studies the problem of an imperfectly aligned intermediate axis torque and derives a design procedure that ensures that a large Δv does not arise.

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Stuck-On Thruster Dynamics

Consider a vehicle with a stuck-on thruster producing a torque approximately about the intermediate principal axis. The resulting motion can be shown[10] to consist of four phases:

1) During the initial phase, motion is quite similar to that encountered in the perfect alignment case: pure spin about the intermediate axis, with only a small initial Δv.

2) During the transition phase, motion becomes significantly different from that of the ideal vehicle. This phase is characterized by a rapid change in angular rates about all three axes and the start of a possible additional build up in linear Δv.

3) During the asymptotic phase, the vehicle converges to spin about either the major or minor principal axis. Angular rate and Δv increase approximately linearly with time. This phase continues until the thruster ceases firing.

4) After thruster cutoff (if the vehicle remains intact) is the final phase of motion, where internal damping will cause the vehicle to converge to spin about the major principal axis.[11]

The main features of the first three phases of the motion (i.e., those for which the thruster is in operation) will now be summarized (see Ref. 10 for further details of the derivation).

Initial/Transition Phases

Suppose the vehicle has inertia matrix $I = \mathrm{diag}(I_1\ I_2\ I_3)$, with $I_1 > I_2 > I_3$, and a thruster producing a torque vector $\boldsymbol{\tau} = \boldsymbol{\tau}_0 + \boldsymbol{\delta\tau}$, where $\boldsymbol{\tau}_0$ is perfectly aligned with the intermediate axis and $\boldsymbol{\delta\tau}$ is small. When the relative rate is defined as $\boldsymbol{\delta\omega} = \boldsymbol{\omega} - \boldsymbol{\omega}_0$, where $\boldsymbol{\omega}_0(t) = (0\ \ \alpha_2 t\ \ 0)^T$ (with angular acceleration $\alpha_2 = \tau_0/I_2$) is the response to the ideal torque $\boldsymbol{\tau}_0$, Euler's equation can be rearranged to give

$$\delta\dot{\boldsymbol{\omega}}(t) \approx I^{-1}[\delta\boldsymbol{\tau} - \alpha_2 t\boldsymbol{e}_2 \times I\delta\boldsymbol{\omega}(t) - \delta\boldsymbol{\omega}(t) \times I_2\alpha_2 t\boldsymbol{e}_2] \tag{1}$$

to first order in $\boldsymbol{\delta\omega}$. When the angular acceleration perturbations are defined as $\{\delta\alpha_i \equiv \delta\tau_i/I_i\}$, the positive dimensionless quantities $\{\kappa_i\}$ as $\kappa_1 \equiv (I_2 - I_3)/I_1$, $\kappa_2 \equiv (I_1 - I_3)/I_2$, and $\kappa_3 \equiv (I_1 - I_2)/I_3$, and integrating, the resulting motion in the initial phase (phase 1) is given as

$$\begin{pmatrix}\delta\omega_1\\ \delta\omega_2\\ \delta\omega_3\end{pmatrix} \approx t\begin{pmatrix}\delta\alpha_1\\ \delta\alpha_2\\ \delta\alpha_3\end{pmatrix} - \frac{1}{3}\alpha_2 t^3\begin{pmatrix}\kappa_1\delta\alpha_3\\ 0\\ \kappa_3\delta\alpha_1\end{pmatrix} \tag{2}$$

The term cubic in t will first become significant (i.e., a linear analysis will break down) when either $\frac{1}{3}\alpha_2\kappa_1\delta\alpha_3 t^3 \approx \delta\alpha_1 t$ or $\frac{1}{3}\alpha_2\kappa_3\delta\alpha_1 t^3 \approx \delta\alpha_3 t$. This, therefore, defines the time at which the complicated transition phase (phase 2) of the motion has begun:

$$t_{\mathrm{trans}} \approx \sqrt{(3/\alpha_2)\cdot\min\{\delta\alpha_1/\kappa_1\delta\alpha_3,\ \delta\alpha_3/\kappa_3\delta\alpha_1\}} \tag{3}$$

Asymptotic Phase

Consider now the fully developed asymptotic phase (phase 3) of the thrusted motion, where high spin rates are experienced. It can be shown[8,10] that a single component of $\boldsymbol{\omega}$ increases approximately linearly with time during this phase, that is, $\boldsymbol{\omega}$ progressively becomes aligned with one of the principal axes of the body. Furthermore, in a result that is quite similar to the well-known one associated with torque-free motion of a rigid body,[12] this asymptotic spin cannot be about the intermediate axis. Assume first that convergence is to spin about the major axis, that is, $|\omega_1| \gg |\omega_2|, |\omega_3|$ for high enough

t. It can then be shown,[8,10] by differentiating the component form of Euler's equation, that the components of ω about the other two axes satisfy

$$\ddot{\omega}_2 \approx -\omega_2 \cdot \kappa_2\kappa_3\omega_1^2, \qquad \ddot{\omega}_3 \approx -\omega_3 \cdot \kappa_2\kappa_3\omega_1^2 \tag{4}$$

The small rates $\omega_2(t)$ and $\omega_3(t)$ are, therefore, approximately sinusoidal, at forced nutation frequency $\omega_{n1} = \omega_1\sqrt{(\kappa_2\kappa_3)}$. The term $\omega_2\omega_3$ on the right-hand side of the first component of Euler's equation, $\dot{\omega}_1 = \{\tau_1 + (I_2 - I_3)\omega_2\omega_3\}/I_1$, is correspondingly cyclic, giving a mean value of $\tau_1/I_1 = \delta\alpha_1$ for $\dot{\omega}_1$. The forced nutation frequency, therefore, becomes

$$\omega_{n1}(t) \approx \delta\alpha_1\sqrt{\kappa_2\kappa_3} \cdot t \tag{5}$$

a quantity that increases linearly with time. The point at which the vehicle enters the asymptotic phase can be defined as that time t_{asym} at which the period of forced nutation (which asymptotically approaches zero) first becomes less than the elapsed time. This reduces to $2\pi/\omega_{n1}(t_{\text{asym}}) = t_{\text{asym}}$, or

$$t_{\text{asym}} \approx \sqrt{\frac{2\pi}{\delta\alpha_1\sqrt{\kappa_2\kappa_3}}} \tag{6}$$

The corresponding results for convergence to a minor axis spin are forced nutation frequency

$$\omega_{n3}(t) \approx \delta\alpha_3\sqrt{\kappa_1\kappa_2} \cdot t \tag{7}$$

and a time constant for entry into the asymptotic phase of

$$t_{\text{asym}} \approx \sqrt{\frac{2\pi}{\delta\alpha_3\sqrt{\kappa_1\kappa_2}}} \tag{8}$$

Whether the vehicle will converge to a major axis or a minor axis spin can be determined by examining the torque Rayleigh quotient[8]

$$R(\boldsymbol{\tau}) = \boldsymbol{\tau}^T I^{-1}\boldsymbol{\tau}/\boldsymbol{\tau}^T\boldsymbol{\tau} \tag{9}$$

This quantity always lies between I_1^{-1} and I_3^{-1}, the minimum and maximum eigenvalues, respectively, of I^{-1}. If $R(\boldsymbol{\tau}) < I_2^{-1}$, convergence will be to a major axis spin; if $R(\boldsymbol{\tau}) > I_2^{-1}$ convergence will be to a minor axis spin. This can be proven[13] by first noting [expanding Eq. (9) by components] that $R(\boldsymbol{\tau}) = I_2^{-1}$ implies that the resulting angular acceleration components satisfy $\delta\alpha_1^2 = \gamma^2\delta\alpha_3^2$, where $\gamma^2 = I_3(I_2 - I_3)/I_1(I_1 - I_2)$. Note that $\omega_1^2 = \gamma^2\omega_3^2$ is the equation of the separatrices[12] of this body, that is, $V \equiv \omega_1^2 - \gamma^2\omega_3^2$ is positive in the region surrounding the major axis and negative around the minor axis. However, from Euler's equation, $\dot{V} = 2(\alpha_1\omega_1 - \gamma^2\alpha_3\omega_3)$. Thus, if V is once positive and $\delta\alpha_1^2 > \gamma^2\delta\alpha_3^2$ [that is, if $R(\boldsymbol{\tau}) < I_2^{-1}$], $\dot{V}$ remains positive and, hence, so does V: Convergence is to a major axis spin. Conversely, if $R(\boldsymbol{\tau}) > I_2^{-1}$, then $\delta\alpha_1^2 < \gamma^2\delta\alpha_3^2$, and convergence is to the minor axis.

Note from the Rayleigh quotient condition that convergence will not necessarily be to that principal axis, major or minor, that is most nearly aligned with the torque direction. The distribution of the principal moments of inertia of the vehicle also plays a central role. (In fact, for the limiting case of an exactly axisymmetric spacecraft, it can be seen that convergence will be to a spin about the symmetry axis for any applied torque that is not precisely normal to this axis.) This has an important practical implication, namely, that the asymptotic rotation axis of the vehicle may be nearly aligned with the direction of the thrust vector of the stuck-on jet. In such a case, even though the vehicle is rotating at a high rate, there will be a relatively large component of linear acceleration that does not cancel over each cycle, namely,

$$a_{\text{asym}} = f_{\text{asym}}/m \tag{10}$$

where m is the vehicle mass and f_{asym} the component of the thrust vector $\boldsymbol{f}$ along the asymptotic spin axis. The resulting net Δv can, therefore, be significant. This is fundamentally different from what occurs in the initial phase of the motion, where rotation takes place about the direction of the applied torque, leading to almost complete cancellation of the applied acceleration over each cycle.

These points will now be illustrated by simulation results for a 50-kg spacecraft, with principal moments $\{I_1, I_2, I_3\} = \{10, 7, 4\}$ kg $\cdot$ m^2, under the action of a stuck-on 0.2-N thruster. This thruster produces a torque that is nominally about the intermediate axis, but is in reality slightly perturbed to be $\boldsymbol{\tau} = (0.0025\ \ 0.1000\ \ 0.0025)^T$ Nm. Equation (9) implies that this torque will lead to convergence to a spin about the minor axis. Figure 1 shows the resulting Δv for two configurations: The solid curve corresponds to the case where the thrust axis of the jet lies along the

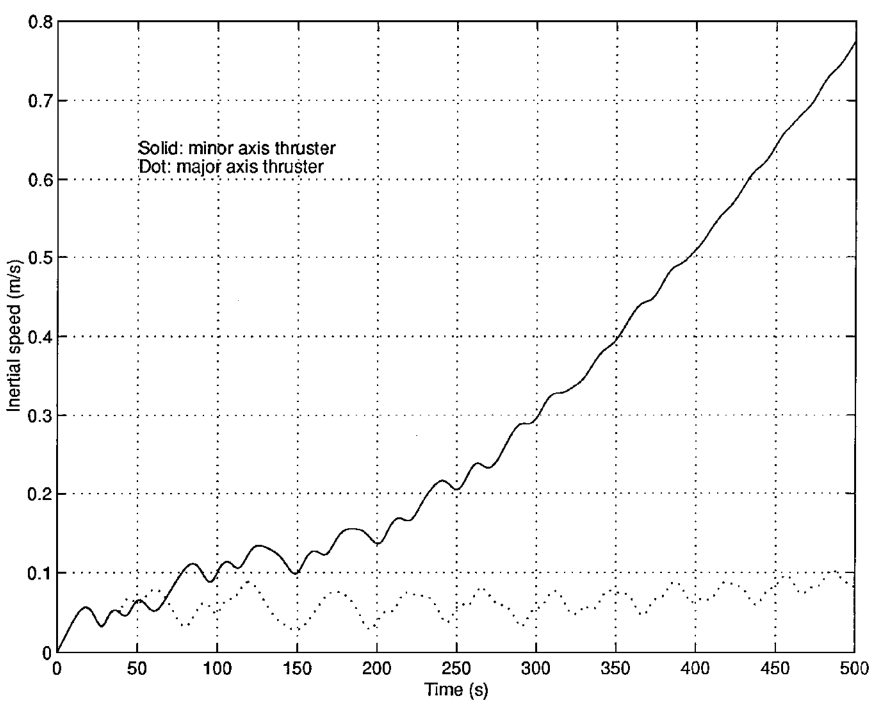

Fig. 1 Δv produced by stuck-on thruster.

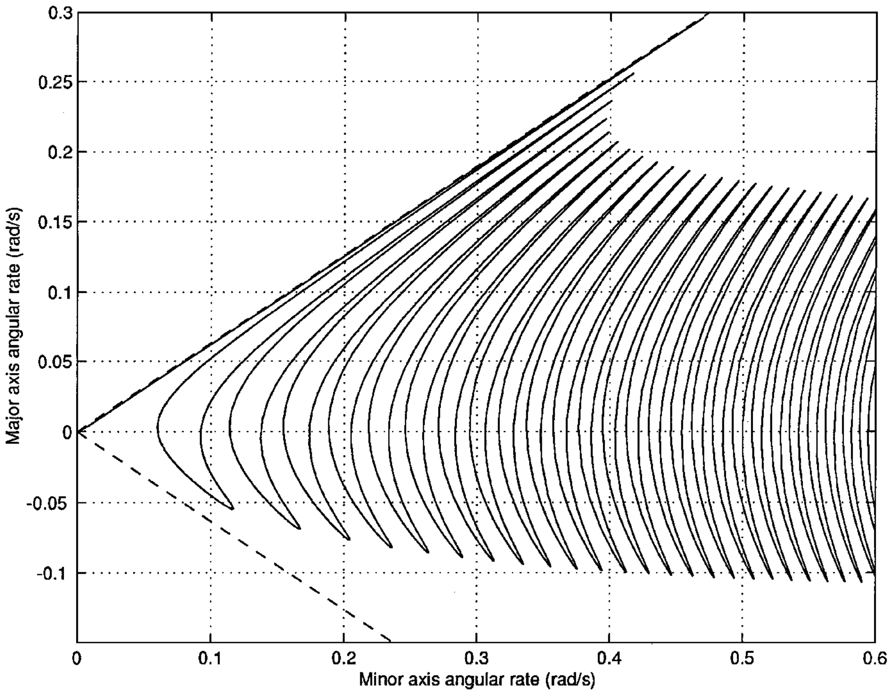

Fig. 2 Evolution of major and minor axis rates.

minor axis (i.e., the asymptotic spin axis), with moment arm along the major axis; the dotted curve corresponds to the alternative geometry of a thruster aligned with the major axis. The greatly increased Δv in the former case is clearly visible, as is the fact that the two plots are virtually identical for around the first 25 s, with the minor axis thruster case reaching an approximately constant linear acceleration by about 200 s. These times agree quite well with the values $t_{\text{trans}} \approx 17$ s and $t_{\text{asym}} \approx 141$ s that are given by Eqs. (3) and (8) for this example. Figure 2 then shows the evolution of the corresponding angular velocity components about the major and minor axes; the separatrices are shown as dashed lines. Note that $(\delta\alpha_1 \;\; \delta\alpha_3) = (0.00025 \;\; 0.00063)$ rad/s for this example, well on the minor axis side of the separatrices. Convergence, therefore, should indeed be to a minor axis spin, as Fig. 2 confirms. Note also the essentially fixed amplitude forced nutation about the major axis, as expected.

Design Procedure to Minimize Stuck-On Thruster Δv

As just noted, a torque that is nominally about the intermediate axis is most naturally produced either by a thruster that fires along the major axis with a moment arm along the minor axis, or vice versa. If a large asymptotic Δv is to be avoided in the former case, convergence must be to a minor axis spin; in the latter, it must be to a major axis spin. This can be accomplished by biasing the torque vector slightly so that $(\delta\alpha_1 \;\; \delta\alpha_3)$ lies on the correct side of the separatrices.

Such a torque bias could be introduced either by rotating the thruster through a small angle, or (probably the preferred approach) by shifting either the position of the thruster relative to the center of mass, or that of the center of mass relative to the thruster, for instance, by means of ballast masses. Introducing a short moment arm component δy along the intermediate axis in this way produces a small torque component about the desired asymptotic spin axis. For example, for the case of a major axis thruster, the resulting torque will be

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{f} = \begin{pmatrix} 0 \\ \delta y \\ z \end{pmatrix} \times \begin{pmatrix} f \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ fz \\ -f\delta y \end{pmatrix} \tag{11}$$

which, indeed, has a component along the minor axis; convergence will, therefore, be to a minor axis spin. Equivalently, the corresponding torque Rayleigh quotient satisfies

$$R(\boldsymbol{\tau}) = \frac{I_2^{-1} z^2 + I_3^{-1} \delta y^2}{z^2 + \delta y^2} = I_2^{-1} + \frac{\delta y^2 \left(I_3^{-1} - I_2^{-1}\right)}{z^2 + \delta y^2} > I_2^{-1} \tag{12}$$

the condition for convergence to a minor axis spin. Note that δy will have no effect on nominal rotational inputs when thrusters are fired in opposing pairs. If the same set of thrusters is also used in like pairs for translational inputs, the moment arm bias will lead to only a small amount of rotational cross coupling in that mode of operation.

A design question that arises with this procedure is that of sizing δy to ensure convergence to the desired principal axis under the worst possible combination of vehicle perturbations. Clearly, δy must be larger than the uncertainty in the intermediate axis component of the thruster position relative to the center of mass. Similarly, the biased torque must remain on the desired side of the separatrices despite all possible degrees of thruster misalignment or variations in vehicle inertia. Inertia variations affect the separatrices in two ways: First, the major and minor principal axes may be rotated; second, the orientation of the separatrices relative to the principal axes may be altered. The latter two effects will now be quantified.

Consider an inertia perturbation matrix $\Delta I = (\Delta I_{ij})$ satisfying the bound $|\Delta I_{ij}| < \delta$ for all i and j. It can then be shown, using standard eigenvector perturbation results,[14] that the angle β through which the major and minor axes may be rotated by the addition of ΔI is bounded by

$$|\sin\beta| \leq \delta/(I_1 - I_3) \tag{13}$$

This inequality implies that the principal axes of bodies with widely separated principal moments of inertia are less sensitive than those of bodies with more closely spaced moments. This is consistent with the extreme sensitivity of the principal axes of near spherically symmetric bodies.[8] For instance, if the nominal moments satisfy $I_1 = 1.02 I_3$, then normalized inertia perturbations (defined to be δ/I_1) of up to 1% can introduce an uncertainty of at most 30 deg in the directions of the principal axes. By contrast, a vehicle with $I_1 = 2I_3$ (i.e., far from spherically symmetric) will only experience a principal axis uncertainty of just over ± 1 deg for the same 1% inertia perturbations.

The effect of ΔI on the orientation of the separatrices (which are at an angle arctan γ relative to the minor axis) can also be quantified. Applying a Gerschgorin disk eigenvalue perturbation analysis[14] yields the bounds $\gamma_{\max} \geq \gamma \geq \gamma_{\min}$ where, to first order in the small δ,

$$\gamma_{\max,\min} \approx \sqrt{\frac{I_3}{I_1} \cdot \frac{(I_2 - I_3) \pm 3\delta(1 + I_2/I_3)}{(I_1 - I_2) \mp 3\delta(1 + I_2/I_1)}} \tag{14}$$

Again, these quantities will be more sensitive if the moments of inertia are close than if they are widely spaced. For a vehicle with nominal inertias satisfying $I_1 = 2I_3$ and $I_2 = 1.5I_3$, and the 1% normalized inertia perturbations considered earlier, arctan γ can vary from 31.8 to 38.7 deg, a total range of 6.9 deg. Using Eqs. (13) and (14) together allows the possible variations in absolute separatrix directions to be quantified as a function of the inertia error bound δ, thus providing the desired insight into sizing δy as a function of δ.

Conclusions

This Note has derived a design procedure that prevents a spacecraft with a stuck-on thruster from experiencing a large net linear acceleration and, hence, a significant perturbation to its orbit. It was shown that such an acceleration is most likely to arise for a thruster that generates a torque that is nominally about the intermediate axis of an asymmetric vehicle, but that is in reality slightly offset from it. The simple technique that was developed to avoid this makes use of a small shift in thruster position. Expressions were derived to allow this shift to be sized to produce robust results despite small uncertainties in the geometry and mass properties of the vehicle.

Acknowledgments

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Experimental Robustness Study of Positive Position Feedback Control for Active Vibration Suppression

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I. Introduction

POSITIVE position feedback (PPF) control was introduced by Goh and Caughey in 1985 to control vibrations of large flexible space structures.[1] A PPF controller has several distinguished advantages as compared to then widely used velocity feedback control laws. It is insensitive to spillover,[2] where contributions from unmodeled modes affect the control of the modes of interest.[3,4] As a second-order low-pass filter, a PPF controller rolls off quickly at high frequencies and is well suited to controlling the lower modes of a structure with well separated modes.[3,5] Because of these advantages, PPF controller along with smart materials, in particular PZT (lead zirconate titanate) type of piezoelectric material, has been applied to many flexible systems to achieve active damping.[6–9] The design of a PPF controller requires the natural frequency of a structure. In practice, the structural natural frequency may not be known exactly or it may vary with time. When the frequency used in the PPF controller is different from that of the structure, the performance of the PPF control will adversely affected. Despite that PPF control is widely researched in literature, robustness study of PPF control when natural frequency is inexactly known is not reported. This motivates the authors to conduct experimental study of robustness of PPF control in active vibration suppression of a smart flexible structure.

II. PPF Control

PPF control requires that the sensor is collocated or nearly collocated with the actuator. In PPF control structural position information is fed to a compensator. The output of the compensator, magnified by a gain, is fed directly back to the structure. The equations describing PPF operation are given as

$$\ddot{\xi} + 2\zeta\omega\dot{\xi} + \omega^2\xi = G\omega^2\eta \tag{1}$$

$$\ddot{\eta} + 2\zeta_c\omega_c^2\dot{\eta} + \omega_c^2\eta = \omega_c^2\xi \tag{2}$$

where ξ is a modal coordinate describing displacement of the structure, ζ is the damping ratio of the structure, ω is the natural frequency of the structure, G is a feedback gain, η is the compensator coordinate, ζ_c is the compensator damping ratio, and ω_c is the frequency of the compensator. For the closed loop being stable, $0 < G < 1$ (Ref. 4).

To illustrate the operation of a PPF controller, assume a single degree-of-freedom vibration of the beam in the form of

$$\xi(t) = \alpha e^{i\omega t} \tag{3}$$

the output of the compensator at the steady state, provided the closed-loop system is stable, will be

$$\eta(t) = \beta e^{i(\omega t - \phi)} \tag{4}$$

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